

## REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

**13[65N30].**—J. TINSLEY ODEN & GRAHAM F. CAREY, *Finite Elements: Special Problems in Solid Mechanics*, Vol. 5, Prentice-Hall, Englewood Cliffs, N. J., 1984, xi+273 pp., 23½ cm. Price \$38.95.

This book contains five chapters written by specialists in finite element methods applied to problems arising in solid mechanics.

The first one is by M. Bernadou, and gives a detailed study of the numerical analysis of thin shell problems. After a brief review of thin shell theories—in both linear and geometrically nonlinear frameworks—the author presents a conforming finite element method with a rigorous error analysis. Several elements are discussed, and the appropriate choice for numerical integration is pointed out. Finally, an application to an arch dam is presented.

The second chapter concerns finite element methods in nonlinear incompressible elasticity and was written by R. Glowinski and P. Le Tallec. There are five sections. The first two are devoted to the formulation of the problem and a description of the notation that is used. The modeling chosen for the numerical solution in the third section includes two unknown fields: the displacement and the pressure (in Lagrange coordinates). The discrete compatibility condition, which *should be* satisfied by the approximate spaces, is pointed out. Then, several examples are exhibited. As a matter of fact, they are the same as those used in fluid mechanics (pressure-velocity formulation). The effective solution of the nonlinear discrete problem via a Newton algorithm is discussed in the fifth section. Finally, an augmented Lagrangian method, based on a three fields approach (displacement, pressure and change of volume ratio), is suggested. This is a tricky procedure which permits one to “localize” the nonlinear constitutive relation for discrete fields. Finally, attractive numerical illustrations are given in the sixth section.

Finite element methods applied to the mechanical study of plastic buckling is the topic of the fourth chapter, which was written by A. Needleman and V. Ivergaard. As a matter of fact, finite element methods are used as a tool, and numerical analysis is very limited in this chapter, in contrast with the other chapters; but a very interesting mechanical discussion, including mechanics of materials, is presented. The numerical tests are very new and presented in an attractive way. In comparison with the other chapters, this chapter contains a lot of numerical information connected with a difficult and important problem.

Chapter 4 by N. Kikuchi and J. T. Oden is devoted to classical contact problems in elastostatics. After a brief recall of Signorini's model, the authors introduce simplifications in order to avoid difficulties connected with the existence of a solution. A finite element method is presented and analyzed for these approximate models. The numerical solution, based on a penalty technique, is also discussed. Finally, a computational test (cylindrical punch on a body) is given.

A new friction law is introduced in Chapter 5. It is due to J. T. Oden and E. B. Pires, who are the authors of this last chapter. In order to avoid difficulties connected with existence of a solution in Signorini's model, a nonlocal law—which can be understood as a regularization of Coulomb's law—is suggested. The local value of the normal stress is approximated by “an average” around the concerned point on the boundary. The variational formulation leads to an existence and uniqueness result for this new model. The final section is devoted to a finite element approximation of the model, for which the same numerical test, as in Chapter 4, is checked.

This book appears to be interesting for a reader who is concerned with mathematical aspects of finite element methods applied to some problems arising in mechanics. Furthermore, its presentation is very good and a homogeneity between the different chapters has successfully been obtained by the editors.

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**14[76S05, 76T05, 65N30].**—GUY CHAVENT & JEROME JAFFRE, *Mathematical Models and Finite Elements for Reservoir Simulation—Single Phase, Multiphase and Multicomponent Flows Through Porous Media*, Studies in Mathematics and its Applications, vol. 17, North-Holland, Amsterdam, 1986, xi+376 pp., 23 cm. Price \$80.00/Dfl. 200.00.

The aim of the book is threefold. First, there is a presentation of several mathematical models related to the reservoir simulation problem. Being unable myself to judge the accuracy and the practical relevance of each model, I can nevertheless say that the exposition is simple, very clear, and accessible, even to people with a rather weak physical background. The various models are presented in a synthetic way, using the new feature of the “global pressure”. Clear hints on various practical situations in which one or another model comes into the game gives one the feeling of being “in contact with the real world”.

A second scope of the book is to provide a rigorous mathematical study of some of the simpler problems. In reading this part, I was rather happy to have spent less time in my life on the study of physics and more on the study of functional analysis. The treatment is indeed very well done, clean, precise and reasonably understandable, provided one has some background in functional analysis.

A third aim of the book is to present some of the new finite element techniques in order to deal with some of these problems. The range of methods that are analyzed